



‘সমানো মন্ত্র: সমিতি: সমানী’

UNIVERSITY OF NORTH BENGAL

B.Sc. Major 1st Semester Examination, 2025

PHYSMAJ101-PHYSICS

MATHEMATICAL PHYSICS-I

Time Allotted: 2 Hours 30 Minutes

Full Marks: 60

The figures in the margin indicate full marks.

GROUP-A

1. Answer any **four** questions from the following: 3×4 = 12
- (a) Find the angle between the surfaces $x^2 + y^2 + z^2 = 9$ and $z = x^2 + y^2 - 3$ at the point $(2, -1, 2)$. 3
- (b) Find the constant ‘a’ such that the vectors $2\hat{i} - \hat{j} + \hat{k}$, $\hat{i} + 2\hat{j} - 3\hat{k}$ and $3\hat{i} + a\hat{j} + 5\hat{k}$ are coplanar. 3
- (c) Find the Wronskian for the following solutions and comment whether the solutions are linearly independent or not: $y_1 = e^{-x} \cos x$ and $y_2 = e^{-x} \sin x$. 2+1
- (d) What is scale factor in co-ordinate system? Find the values of it for spherical co-ordinate system. 1+2
- (e) State the Existence and Uniqueness Theorem for initial value problems.
- (f) Evaluate $\iint_s \vec{r} \cdot \hat{n} \, ds$ where s is a closed surface.

GROUP-B

Answer any **four** questions from the following

6×4 = 24

2. (a) Prove that the vector $\vec{A} = 3y^4 z^2 \hat{i} + 4x^3 z^2 \hat{j} - 3x^2 y^2 \hat{k}$ is solenoidal. 2
- (b) Show that $\vec{\nabla} \cdot (\vec{\nabla} r^n) = n(n+1)r^{n-2}$, where $r = \sqrt{x^2 + y^2 + z^2}$. 4
3. (a) Find the degree and order of the differential equation $\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2} = \frac{d^2 y}{dx^2}$. 1+1
- (b) Solve: $\sin x \frac{dy}{dx} = \tan^3\left(\frac{x}{2}\right)$ 4
4. Represent the vector $\vec{A} = 2y\hat{i} - z\hat{j} + 3x\hat{k}$ in cylindrical coordinates. Thus determine A_r , A_θ and A_ϕ . 5+1
5. (a) State Taylor’s Theorem. 1
- (b) Expand $\cos z$ in a Taylor series about $z = \frac{\pi}{4}$. 3
- (c) Expand $\sqrt[3]{2-4x}$ using the Binomial series upto the term in x^3 . 2

6. (a) Solve $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 5y = 0$. Given, $y = 2$ and $\frac{dy}{dx} = \frac{d^2y}{dx^2}$ when $x = 0$. 4
- (b) If a force $\vec{F} = 2x^2y\hat{i} + 3xy\hat{j}$ displaces a particle in the xy plane from $(0, 0)$ to $(1, 4)$ along a curve $y = 4x^2$. Find the work done. 2
7. (a) If $\vec{r}(t)$ is a constant vector, then prove that $\vec{r}(t)$ is always perpendicular to $\frac{d\vec{r}}{dt}$. 2
- (b) Prove that $\vec{a} \times (\vec{b} \times \vec{c}) + \vec{b} \times (\vec{c} \times \vec{a}) + \vec{c} \times (\vec{a} \times \vec{b}) = \vec{0}$. 2
- (c) Show that the divergence of the curl of a vector is always zero. 2

GROUP-C

Answer any two questions from the following

12×2 = 24

8. (a) Find the unit vectors of a spherical polar coordinate system and show that the system is orthogonal and right handed. 3+1+2
- (b) Express $\vec{\nabla}\phi$, $\vec{\nabla} \cdot \vec{A}$ and $\vec{\nabla} \times \vec{A}$ in cylindrical coordinates. 2+2+2
9. (a) State and prove the Green's Theorem. 2+3
- (b) Verify Green's Theorem in the plane for $\oint (3x^2 - 8y^2)dx + (4y - 6xy)dy$, where c is the boundary of the region defined by $y = \sqrt{x}$ and $y = x^2$. 5
- (c) Evaluate using Stokes' theorem $\oint (yz dx + zx dy + xy dz)$ where c is the curve $x^2 + y^2 = 1$, $z = y^2$. 2
- 10.(a) Find the complete solution of $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = xe^{3x} + \sin 2x$. 5
- (b) Solve the differential equation $\frac{d^2y}{dx^2} + a^2y = \frac{a^2R}{\rho}(l-x)$ where a, R, ρ and l are constants. Given, $y = 0, \frac{dy}{dx} = 0$ at $x = 0$. 5
- (c) Find integrating factor for the differential equation $\frac{dy}{dx} + xy = x^3y^3$. 2
- 11.(a) A fluid motion is given by $\vec{V} = (y \sin z - \sin x)\hat{i} + (x \sin z + 2yz)\hat{j} + (xy \cos z + y^2)\hat{k}$ Show that the motion is irrotational. Hence, find the velocity potential. 2+2
- (b) Consider a vector $\vec{A} = \hat{i}Ax + \hat{j}Ay + \hat{k}Az$. Now the coordinate system is rotated about z axis in anticlockwise direction through an angle θ . In the rotated frame the vector is given by $\vec{A}' = \hat{i}Ax' + \hat{j}Ay' + \hat{k}Az'$. Find the relation between the components of $\vec{A}$ and $\vec{A}'$. 4
- (c) Show that the scalar product of two vectors is invariant under coordinate system rotation. 2
- (d) Prove that $\vec{\omega} = \frac{1}{2}\vec{\nabla} \times \vec{v}$, where $\vec{v}$, $\vec{\omega}$ are linear and angular velocity respectively. 2

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UNIVERSITY OF NORTH BENGAL

B.Sc. Major 1st Semester Examination, 2025

PHYSMAJ102-PHYSICS

MECHANICS

Time Allotted: 2 Hours 30 Minutes

Full Marks: 60

The figures in the margin indicate full marks.

GROUP-A

1. Answer any **four** questions from the following: 3×4 = 12
- (a) Write down Galilean Transformation equations. Show that under this transformation, Newton's second law remains invariant. 1+2
- (b) Show that in a conservative force field, the total energy of a particle is conserved. 3
- (c) Find the moment of inertia of a solid disc of mass M and radius R about an axis passing through its centre and perpendicular to its plane. 3
- (d) Trajectory of a particle of unit mass is given by the radius vector $\vec{r} = a \cos \omega t \hat{i} + b \sin \omega t \hat{j} + c \hat{k}$, where a , b and c are constants. Calculate the angular momentum of the particle about the origin. Show that it is constant along $\hat{k}$. 1½+1½
- (e) Define Young's modulus, Bulk modulus and modulus of rigidity. Write down their mathematical expression. 1+1+1
- (f) State Kepler's 3rd law of planetary motion and deduce it from Newton's law of gravitation. 1+2

GROUP-B

Answer any **four** questions from the following

6×4 = 24

2. (a) Explain what do you mean by equilibrium. Define stable and unstable equilibrium. 1+2
- (b) The potential energy $V(x)$ of a particle is given by 3
- $$V(x) = 3x^4 - 8x^3 - 6x^2 + 24x$$
- Determine the point of stable and unstable equilibrium.
3. Find the velocity and acceleration of a particle in spherical polar coordinates. 6
4. (a) Calculate the moment of inertia of a thin uniform rod about an axis through its centre and perpendicular to its length. 3
- (b) State parallel axes theorem and calculate the moment of inertia of a rectangular plate about an axis passing through one corner and perpendicular to its plane. 3
5. (a) Why is the gravitational potential always negative? 1
- (b) Determine the gravitational potential at a point (i) outside, (ii) inside and (iii) on the surface of a hollow and thin spherical shell. 5
6. (a) For glass, the Young's Modulus and the Poisson's ratio have the values $7.2 \times 10^{10} \text{ N/m}^2$ and 0.25 respectively, calculate the modulus of rigidity of glass. 3
- (b) 'Steel is more elastic than rubber' — Do you agree with the statement? Give reason. 3

7. (a) Show that $\vec{N} = \frac{d\vec{L}}{dt} = \vec{r} \times \vec{F}$, where the symbols have their usual meaning. 3
- (b) Find out the potential energy function for the following conservative force, 3
- $$\vec{F} = \hat{i}(2xy^2 - yz) + \hat{j}(2x^2y - xz) - \hat{k}xy$$

GROUP-C

Answer any two questions from the following

12×2 = 24

8. (a) Describe the principle of rocket motion. Establish the following relation for a rocket 6+6

$$v(t) = v_0 + u \ln \left(\frac{m_0}{m(t)} \right)$$

where v_0 is the initial speed, u is constant fuel ejection speed, m_0 is the initial total mass, and $m(t)$ is the total mass of the rocket after time interval t .

- (b) A sphere of radius ' R ' has a cavity of radius ' r ' from its centre. Find the moment of inertia of the sphere about its own axis passing through its centre.
9. (a) If Y , κ and η represent Young's Modulus, Bulk Modulus and Modulus of Rigidity respectively, prove the following relation, 5+2+2+3

$$Y = \frac{9\eta\kappa}{3\kappa + \eta}$$

- (b) If the diameter of a wire is made double keeping the length same, how would the ' Y ' change? Justify your answer.
- (c) Prove that Poisson's ratio lies between -1 and $1/2$.
- (d) Calculate the Poisson's ratio for a material, given $Y = 12.25 \times 10^{10} \text{ N/m}^2$ and $\eta = 4.55 \times 10^{10} \text{ N/m}^2$.
- 10.(a) For a particle moving under central force, show that 3+2
- (i) the angular momentum of the particle about the centre of force is conserved and
- (ii) the particle moves in a plane.
- (b) What is escape velocity? Derive an expression for it for the planet earth in terms of ' g ' and the radius ' R ' of the earth. 1+3
- (c) A planet revolves around a star in an elliptic orbit. The ratio of the farthest distance to the closest one of the planet from the star is 4. Find the ratio of the kinetic energies of the planet at the farthest to the closest positions. 3
- 11.(a) Show how a fictitious force arises in a non-inertial frame which is moving with a constant acceleration in a given direction with respect to a fixed frame. 4+4+4
- (b) Calculate the deviation from its vertical path due to Coriolis force for a particle falling from a height 100 m at 45°N latitude.
- (c) Determine whether the following forces are conservative or not.
- (i) $\vec{F}_1 = \frac{\alpha}{r^4}x\hat{i} + \frac{\alpha}{r^4}y\hat{j} + \frac{\alpha}{r^4}z\hat{k}$ where α is a constant.
- (ii) $\vec{F}_2 = (3z + 5y)\hat{i} + (5x + 2z)\hat{j} + (2y + 3x + 4z)\hat{k}$.

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